

## Landau levels

Consider motion in a strong magnetic field

$\omega_c \tau \gg 1$  (regime of "Quantum Hall" effect)

$\omega_c = \frac{eB}{mc}$  - cyclotron frequency

The Hamiltonian of the system

$$\hat{H} = \frac{1}{2m} \left( \hat{\vec{p}} - \frac{e}{c} \vec{A} \right)^2$$

Pick the gauge:

$\vec{A} = (0, Hx, 0)$  - Landau gauge

( $\vec{A} = (-Hy, 0, 0)$  - also Landau gauge)

The Ham-n then is translationally invariant in the  $y$  direction

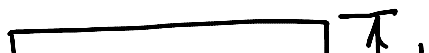
$$\Psi = e^{i p_y y} \chi(x)$$

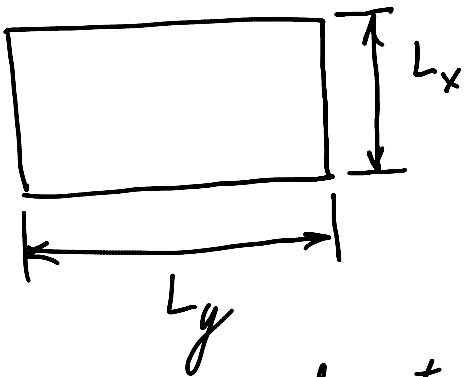
$$\left[ -\frac{1}{2m} \partial_x^2 + \frac{1}{2m} \left( p_y - \frac{e}{c} Bx \right)^2 \right] \chi = E \chi$$

Change variables:  $p_y - \frac{e}{c} Bx \rightarrow -\frac{e}{c} B\tilde{x}$

$$\tilde{x} = x - \frac{c}{eB} p_y$$

To determine in what ranges all variables may change, consider a system of specific shape





$\tilde{x}$  varies on characteristic scales shorter than the sizes of the system

$x$  may change from  $-\frac{L_x}{2}$  to  $+\frac{L_x}{2}$

Then  $p_y$  will vary between  $-\frac{eB}{2c}L_x$  and  $+\frac{eB}{2c}L_x$

The total number of values of  $p_y$  is

$$\frac{|e|B}{2\pi c} L_x L_y \equiv \gamma = \frac{|e|B}{2\pi c} S = \frac{BS}{\varphi_0}$$

The Ham-n after the change of variables  $x \rightarrow \tilde{x}$ :

$$\left[ -\frac{1}{2m} \partial_{\tilde{x}}^2 + \frac{e^2 B^2}{2m c^2} \tilde{x}^2 \right] \chi = E \chi$$

The Ham-n is  $p_y$  independent, but all levels acquire an extra degeneracy of  $\gamma$

$$\left( -\frac{1}{2m} \partial_{\tilde{x}}^2 + \frac{m\omega_c^2}{2} \tilde{x}^2 \right) \chi = E \chi$$

- the Ham-n of a harmonic oscillator

$$E_n = \hbar \omega_c \left( n + \frac{1}{2} \right) \quad \text{- Landau levels}$$

$$E_n = \hbar \omega_c \left( n + \frac{1}{2} \right) \quad - \text{Landau levels}$$

each has degeneracy  $\gamma = \frac{BS}{\phi_0}$

A characteristic lengthscale

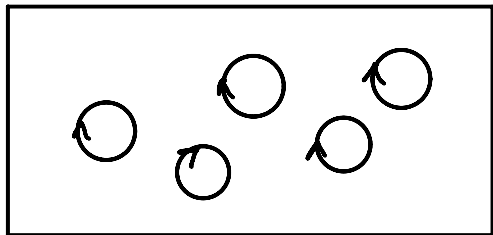
$$l_B = \sqrt{\frac{\hbar}{m\omega_c}} = \sqrt{\frac{\hbar c}{|e|B}}$$

---

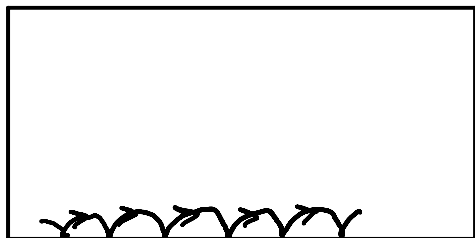
The states in the bulk of the system carry no current

$$\frac{\partial E_n}{\partial p_x} = \frac{\partial E_n}{\partial p_y} = 0$$

May be expected to be an insulator; interplay with disorder will be important though



The edge states are different though...



an effectively 1D conducting channel

The energy of a state vs. mean distance to the wall

